

# Long-term care policy with nonlinear strategic bequests\*

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## Abstract

We study the design of long-term care (LTC) policies when children differ in their cost of providing informal care. Parents do not observe this cost, but they can commit to a “bequests rule” specifying a transfer conditional on the level of informal care. Care provided by high-cost children is distorted downwards in order to minimize the rent of low-cost ones. Social LTC insurance is designed to maximize a weighted sum of parents’ and children’s utility. The optimal *uniform* public LTC provision strikes a balance between insurance and children’s utility. Under decreasing absolute risk aversion less than full insurance is provided to mitigate the distortion on informal care which reduces children’s rents. A *nonuniform* policy conditioning LTC benefits on bequests provides full insurance even against the risk of having children with a high cost of providing care. Quite surprisingly the level of informal care induced by the optimal (uniform or nonuniform) policy always *increases* in the children’s welfare weight.

**JEL classification:** H2, H5, I13, J14.

**Keywords:** Long-term care, informal care, strategic bequests, asymmetric information.

# 1 Introduction

Old age dependence and the need for long-term care (LTC) it brings about represent a major societal challenge in most developed countries. Due to population ageing the number of dependent elderly with cognitive and physical impairments will increase dramatically during the decades to come. LTC needs start to rise exponentially from around the age of 80 years-old. The number of persons aged 80 years and above is growing faster than any other segment of the population. As a consequence, the number of dependent elderly at the European level (EU-27) is expected to grow from about 21 million people in 2007 to about 44 millions in 2060 (EC, 2009). Similar trends are in the forecast for the US.<sup>1</sup>

Dependence represents a significant financial risk of which only a small part is typically covered by social insurance.<sup>2</sup> Private insurance markets are also thin.<sup>3</sup> Instead, individuals rely on their savings or on informal care provided by family members. Currently the family is the main provider and informal care representing roughly 2/3 of total care (Norton, 2000). Informal provision has no direct bearing on public finances, but it is not available to everyone. Whether this solidarity is sustainable at its current level is an important question. Sources of concern are numerous. The drastic change in family values, the increasing number of childless households, the mobility of children, the increasing labor participation of women are as many factors explaining why the number of dependent elderly who cannot count on family solidarity, at least not for the full amount of care they need, is increasing. Furthermore, it is not clear whether the important role played by informal care is desirable. Its real costs are often “hidden”. In particular, it may indeed impose on caregivers a significant burden which is both

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<sup>1</sup>See Cremer *et al.*, (2012) or Grabovski *et al.* (2012) for extensive overviews of the LTC need projections.

<sup>2</sup>For instance average daily costs of nursing homes in the US in 2017 is \$235 (but is typically closer to \$400 in the Northeast); see <https://www.payingforseniorcare.com/longtermcare/paying-for-nursing-homes.html>

The average stay in a nursing home is 835 days, according to the National Care Planning Council, which brings the average total cost to about \$200,000 (and twice that amount in some states).

<sup>3</sup>The literature has presented a number of explanations for this “LTC insurance market puzzle”, including adverse selection (Finkelstein and McGarry, 2006) and the parent’s preference for informal care (Pauly, 1990).

financial and psychological.<sup>4</sup>

In a nutshell, the current situation does not appear to be efficient as it leaves some elderly without proper care and often imposes a considerable burden on caregivers. This creates a potential role for public intervention through social LTC provision or insurance. However, the public LTC policy will interact with informal care, and more generally with exchanges within the family; consequently, policy design has to account for the induced changes in transfers within the family.

Informal care can be motivated by some form of altruism, result from implicit exchanges, be “imposed” by social norms, or results from a combination of these factors. Understanding the foundation of informal care is very important in order to predict how family assistance will react to the emergence of private or public schemes of LTC insurance.

In this paper we consider a setting where children can provide informal care to dependent parents, and intergenerational exchanges are based on a care vs. bequest (or gift) exchange.<sup>5</sup> This does not rule out that children display some degree of altruism and would provide *some* care even without financial incentives. However, parent’s may want to “buy” more than this minimum level.<sup>6</sup> We do not *explicitly* introduce altruism, but assume that it indirectly affects the children’s cost of providing care. More precisely, children differ in their preferences, which determine their cost (disutility) of providing informal care. This heterogeneity reflects differences in the children’s degree of altruism, the strength of family bonds and possibly a number of other factors such as their proximity, working situations, and marital status.

Our framework is inspired by the strategic bequest approach.<sup>7</sup> We assume that parents can commit to a bequest rule specifying a transfer, gift or bequests, conditional on the level of informal care provided.<sup>8</sup> However, our setting crucially differs from the

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<sup>4</sup>On the costs borne by caregivers, see Colombo *et al.* (2011) or Coe and Van Houtven (2009).

<sup>5</sup>We revisit this assumption and discuss its empirical relevance in the conclusion.

<sup>6</sup>See Bernheim *et al.* (1985) who explicitly consider some degree of altruism for the children.

<sup>7</sup>See for instance Kotlikoff and Spivak (1981), Bernheim *et al.* (1985).

<sup>8</sup>To be more precise, Bernheim *et al.* (1985) do not assume commitment in an *ad hoc* way, but show how it can be achieved endogenously as long as there are at least two potential heirs. Most of the subsequent literature cut short on this issue and assumes commitment from the outset.

conventional model in that parents do not perfectly know their children’s preferences, and their cost of providing informal care. Absent of full information, parents can no longer allow extract from their children the full surplus generated by the exchange. Whether this lack of knowledge is due to uncertainty or to asymmetric information is of no relevance in our setting. Even though parents can use nonlinear rules to screen for the children’s cost parameter (once this has been realized), they will have to leave a positive rent to some of the children.

In the *laissez-faire*, the help provided by high-cost children is distorted downwards in order to mitigate the rent enjoyed by the low-cost ones. Parents are not insured against the risk of dependence, nor against the risk that their children have a high cost of providing care.

We then introduce social LTC in this setting. It is designed to maximize a weighted sum of parents’ and children’s utilities. In other words we explicitly account for the wellbeing of caregivers. This further differentiates our paper from most of the literature, which has to a large extent concentrated on parents’ welfare.<sup>9</sup> To concentrate on LTC policies rather than deal with the more general issue of intergenerational allocation of resources, we assume throughout the paper that the government cannot use lump-sum taxes and transfers to redistribute between parents and children.<sup>10</sup>

In the first part of the paper we consider a *uniform* social LTC policy. It provides to *all* dependent individuals a given LTC transfer, which is financed by a uniform lump-sum tax. We show that this policy affects distortions of informal care and thus the distribution of rents between parents and children. The optimal policy then involves a tradeoff between providing insurance to parents and enhancing the utility of the caregivers. In the absence of informal care, social LTC insurance should fully insure the risk of dependence. With informal care and strategic bequests, the optimal public LTC policy depends on the attitude towards risk of parents. Under DARA (decreasing absolute risk aversion) preferences, public LTC insurance exacerbates the distortion of informal

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<sup>9</sup>Barigozzi *et al.* (2017) also account for the welfare of caregivers, but consider a different type of family exchanges based on cooperative bargaining.

<sup>10</sup>In our setting with quasi-linear preferences allowing for such taxes transfers would yield corner solutions where the entire surplus (and not just that generated by care for transfer exchanges) would be given to a single generation. See footnote 24 for a more detailed discussion.

care. Better insurance coverage makes dependent parents less reliant on informal care, so that distorting down informal care is not too costly for them. Consequently, under DARA, the optimal public LTC policy provides less than full insurance in order to mitigate the distortion of informal care and the reduction in (low-cost) children's utility it brings about. The opposite is true under IARA (increasing absolute risk aversion) preferences, in which case the government should provide more than full insurance against the risk of dependence in order to minimize distortions.

In the second part of the paper we consider *nonlinear* policies, where transfers from parents to children are publicly observable and LTC benefits can be conditioned on bequests (or gifts). The LTC policy can then screen for the children's cost parameter, even when the level of informal care is observable only to parents. The underlying problem presents methodological challenges because we have to deal with a "nested" principal-agent problem. While the policy can screen for the children's cost of providing care, this is done only indirectly via the parents. The latter do not observe their children's cost of providing care either but since they observe informal care, they have superior information.

We show that, while with a uniform policy the results crucially depend on parents' risk aversion, this is no longer true when the policy is restricted by informational considerations only and can be nonuniform. In that case the available policy instruments are sufficiently powerful to ensure that parents are always fully insured, even against the risk of having high-cost children. And since they are fully insured, risk aversion no longer matters. Even more strikingly, the tradeoff between the provision of insurance to parents and the concern for the welfare of the caregivers which drives the results for a uniform policy is not longer relevant when nonuniform policies are considered.

One surprising result regarding the impact of social LTC insurance on informal care emerges in both cases. In most of the literature on LTC, crowding out of informal care by public care is considered a serious problem. It makes the provision of social LTC insurance more expensive and possibly even ineffective, when there is full crowding out. If one accounts for the wellbeing of caregivers the impact of crowding out is more complex. One might conjecture that it is "bad" for parents, but "good" for children,

since it reduces the cost implied by care provision. However, this conjecture does not stand under closer scrutiny, and is proven wrong or misleading by our results. This is because in our setting children are paid for their care through gifts or bequests. Under full information, when care decreases, their compensation is reduced to keep their utility constant. Crowding out is then irrelevant. Under asymmetric information, on the other hand, crowding out will reduce the rents of some of the children and thus effectively decrease their utility, while it increases parents' welfare. Consequently, the concern for the wellbeing of caregivers does *not* imply that LTC policy should aim at reducing informal care. Quite the opposite, we show that the optimal level of informal care increases if their weight in the social welfare function increases.

The paper is organized as follows. We present the model in Section 2. We study the uniform and the nonlinear LTC policies, in Section 3 and 4, respectively.

## 2 Model

Consider a generation of identical parents. When old they face the risk of being dependent with probability  $\pi$ , while they are independent and healthy with probability  $(1 - \pi)$ . When young they each have one child, they earn an exogenous labor income  $w\bar{T}$  of which they save  $s$ . They have preferences over consumption when young,  $c \geq 0$ , consumption when old and healthy,  $d \geq 0$ , and consumption, including LTC services, when old and dependent,  $m \geq 0$ . Their preferences are quasilinear in consumption when young. Risk aversion is introduced through the concavity of second period state dependent utilities. The parents' expected utility is given by

$$EU = w\bar{T} - s + (1 - \pi)U(s) + \pi E[H(m)],$$

with  $m = s + a - \tau(a)$ , where  $a \in [0, a^{max}]$  is informal care provided by children, while  $\tau(a)$  is a transfer (bequest or gift) from parents to children.<sup>11</sup> One can think of  $m$  as total care, that is informal care  $a$  plus formal care  $s - \tau(a)$ , the price of which is normalized to one. We assume that the transfer can be conditioned on informal care and assume

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<sup>11</sup>The assumption that there is an upper bound on informal care is quite natural. Furthermore it is convenient for technical reasons.

that parents can commit to this bequest rule. This is in line with the strategic bequest literature. However, our model differs from the traditional literature on exchange based intergenerational transfers in that we assume that at the “contracting stage”, when they set the bequest rule parents may not perfectly observe their child’s preferences and in particular their cost of providing care.<sup>12</sup> The children’s heterogeneity in cost is represented by a parameter  $\beta$  which is not publicly observable, including to parents. This parameter captures a number of factors that may affect the provision of informal care without being perfectly observed by parents, such as the strength of family ties or the children’s degree of altruism. Assume that  $\beta$  is distributed over  $\{\underline{\beta}, \bar{\beta}\}$ . In other words,  $\beta$  can only take two values. The low one  $\underline{\beta}$  occurs with probability  $\lambda \in ]0, 1[$ , while the high level  $\bar{\beta}$  occurs with probability  $(1 - \lambda)$ .

Children’s cost of providing care  $a$  to their parents is given by  $v(a, \beta)$ , with  $v_a > 0$ ,  $v_\beta < 0$ ,  $v_{aa} > 0$ ,  $v_{a\beta} < 0$ , where subscripts denote partial derivatives. The cost is increasing and convex in the level of informal care. It *decreases* in  $\beta$ , which amounts to saying that  $\bar{\beta}$  is the “good type” for whom providing care is less costly. Furthermore  $v_{a\beta} < 0$  implies that the *marginal* cost of informal care also decreases with  $\beta$ .

The children’s utility from helping their parents in case of dependence is

$$U_k = c_k - v(a, \beta) \geq 0,$$

where children’s consumption  $c_k = \tau(a)$ , the transfer from their parents.

Children choose  $a$  to maximize  $U_k$ . The first order condition is

$$\tau'(a) = v_a(a, \beta), \tag{1}$$

and the solution to this problem is denoted  $a(\beta)$ . Observe that  $a$  also depends on  $\tau(\cdot)$ , exactly as labor supply depends on the tax function in a Mirrleesian-type optimal income tax problem.

Anticipating their children’s behavior but not observing  $\beta$ , parents choose  $s$  and

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<sup>12</sup>In other words, we assume that parents decide on their bequest and gift dispositions at a time when they are still in good health and in a position to do so. Recall that we are concentrating on old age dependency linked mainly to cognitive affections like Alzheimer’s or other forms of dementia.



$\tau(a)$  to maximize their expected utility given by

$$EU = w\bar{T} - s + (1 - \pi)U(s) + \pi E_{\beta} [H(s + a(\beta) - \tau(a(\beta)))].$$

To solve this problem we consider the equivalent mechanism design problem where parents choose  $a(\beta)$  and  $\tau(\beta)$  to maximize

$$EU = w\bar{T} - s + (1 - \pi)U(s) + \pi E_{\beta} [H(s + a(\beta) - \tau(\beta))]$$

subject to the relevant participation constraints, as well as the incentive constraints. These constraints come about when the parents do not observe  $\beta$ . They will be stated and explained below.

Within this framework, we study the public provision of LTC benefits financed by a lump-sum tax on parents' first-period consumption. In addition, intergenerational transfers  $\tau$  may be taxed or subsidized. The policy is determined to maximize social welfare, which is given by a weighted sum of parents' and children's expected utilities. Parents' weight is  $\alpha \in (0, 1)$ , while children's utility is weighted by  $1 - \alpha$ .

The timing is as follows. The LTC policy is decided upon in stage 1, before parents and children make their decisions. In stage 2, parents choose their level of savings  $s$  and commit to the bequest rule  $\tau(a)$ . In stage 3, the dependence status of the parents is realized and children choose  $a$  according to (1).

While we concentrate on the asymmetric information case, we start by considering the full information benchmark, that is the solution parents can achieve when they observe their children's cost of providing care. We also characterize the optimal LTC policy for this benchmark.

### 3 Uniform LTC benefit

In this section, we restrict the policy to a uniform transfer  $g$  to dependent parents financed by a lump-sum tax. In other words, we consider universal public provision (or subsidization) of LTC. Such a policy is clearly suboptimal if the government can condition transfers on bequests.<sup>13</sup> However, it is relevant in practice, since the government

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<sup>13</sup>In Section 4 we will consider the case where  $g$  can be conditioned on bequests, so that the government can screen for different levels of  $\beta$ .

may not be able to observe bequests. Furthermore, even if the government was able to observe bequests, it may be politically infeasible to condition  $g$  on  $\tau$ . In this section, we first characterize the optimal policy when parents can observe their children's type. We then turn to the asymmetric information case.

### 3.1 Full information benchmark

Parents choose  $s$  and commit to a bequest rule *ex ante*, that is before the state of dependence and  $\beta$  are realized and observed. Since  $\beta$  takes only two values, it is convenient to introduce the following notation

$$\begin{aligned} \tau(\bar{\beta}) &= \bar{\tau}, & ; & & \tau(\underline{\beta}) &= \underline{\tau}, \\ a(\bar{\beta}) &= \bar{a}, & ; & & a(\underline{\beta}) &= \underline{a}, \\ m(\bar{\beta}) &= \bar{m}, & ; & & m(\underline{\beta}) &= \underline{m}. \end{aligned}$$

Using this notation the parent's problem can be written as

$$\begin{aligned} \max_{\bar{a}, \underline{a}, \bar{\tau}, \underline{\tau}, s} \quad & w\bar{T} - \pi g - s + (1 - \pi) U(s) + \pi [\lambda H(s + \underline{a} - \underline{\tau} + g) + (1 - \lambda) H(s + \bar{a} - \bar{\tau} + g)] \\ \text{s.t.} \quad & \underline{\tau} - v(\underline{a}, \underline{\beta}) \geq 0, & (2) \end{aligned}$$

$$\bar{\tau} - v(\bar{a}, \bar{\beta}) \geq 0. \quad (3)$$

where  $\pi g$  is the lump-sum tax levied to finance the expected cost of social LTC transfers. Conditions (2) and (3) represent the children's participation constraint. While children take the bequest rule  $\tau(a)$  as given, they have the option not to exchange with their parents: in this case there will be no care and no transfer and the children's utility is an exogenously given constant which without loss of generality is normalized to zero.

Under full information, the parent can extract all the surplus, and both participation constraints are binding.<sup>14</sup> Then, substituting for  $\underline{\tau}$  and  $\bar{\tau}$  from (2) and (3) the parent's problem can be rewritten as

$$\begin{aligned} \max_{\bar{a}, \underline{a}, s} \quad & P^f = w\bar{T} - \pi g - s + (1 - \pi) U(s) + \pi [\lambda H(s + \underline{a} - v(\underline{a}, \underline{\beta}) + g) \\ & + (1 - \lambda) H(s + \bar{a} - v(\bar{a}, \bar{\beta}) + g)]. \end{aligned}$$

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<sup>14</sup>When a participation constraint is not binding parents can increase the corresponding  $a$  and/or decrease  $\tau$ , thereby increasing their expected utility.

The first-order conditions (FOC) with respect to the remaining choice variables are given by

$$\begin{aligned}\frac{\partial P^f}{\partial \bar{a}} &= (1 - \lambda)H'(\bar{m})[1 - v_a(\bar{a}, \bar{\beta})] = 0, \\ \frac{\partial P^f}{\partial \underline{a}} &= \lambda H'(\underline{m})[1 - v_a(\underline{a}, \underline{\beta})] = 0, \\ \frac{\partial P^f}{\partial s} &= -1 + (1 - \pi)U'(s) + \pi[\lambda H'(\underline{m}) + (1 - \lambda)H'(\bar{m})] = 0.\end{aligned}\quad (4)$$

The first two conditions imply<sup>15</sup>

$$1 = v_a(\bar{a}, \bar{\beta}) = v_a(\underline{a}, \underline{\beta}), \quad (5)$$

which is quite intuitive. Under full information parents have to compensate children exactly for their utility cost of informal care. Consequently, they equalize marginal costs to marginal benefits, which are equal to one. Not surprisingly, this implies  $\bar{a} > \underline{a}$  and  $\bar{m} > \underline{m}$ : low-cost children provide more informal care and their parents enjoy a larger amount of total care,  $m$ , in case of dependence. To decentralize this solution parents must then choose  $\tau$  so that  $\tau'(\bar{a}) = \tau'(\underline{a}) = 1$ . Observe that neither  $\bar{a}$  nor  $\underline{a}$  depend on  $g$ . Consequently, a uniform  $g$  can never achieve full insurance for the risk associated with the uncertainty of  $\beta$ ; parents with low-cost children are always better off.

We now turn to the government's problem, which is given by

$$\begin{aligned}\max_g \quad G^f &= w\bar{T} - \pi g - s + (1 - \pi)U(s) + \pi [\lambda H(s + \underline{a} - v(\underline{a}, \underline{\beta}) + g) \\ &\quad + (1 - \lambda)H(s + \bar{a} - v(\bar{a}, \bar{\beta}) + g)],\end{aligned}\quad (6)$$

where  $s$ ,  $\underline{a}$  and  $\bar{a}$  are the solutions to the parents' problem for any given  $g$ . Since the parents have full information, the children's utility will be zero no matter what. Consequently, the relative weight of children in social welfare is of no relevance and we can just as well neglect this term in the welfare function.

Using the envelope theorem (for the induced effect on  $s$ ) and recalling that the levels of  $a$  do not depend on  $g$  we have<sup>16</sup>

$$\frac{\partial G^f}{\partial g} = -\pi + \pi[\lambda H'(\underline{m}) + (1 - \lambda)H'(\bar{m})] = 0.$$

<sup>15</sup>We assume that the solution is interior, that is to say that  $\underline{a} > 0$  and  $\bar{a} < a^{max}$ .

<sup>16</sup>We assume that the second order condition holds.

Not surprisingly, this condition equalizes marginal costs and benefits of  $g$ . Combining this condition with the parent's FOC with respect to  $s$  in (4) yields

$$U'(s) = [\lambda H'(\underline{m}) + (1 - \lambda)H'(\overline{m})] = E_\beta[H'(m)] = 1. \quad (7)$$

This condition states that the three possible uses of (first-period) income, direct consumption  $c$ , deferred consumption  $s$ , and LTC insurance,  $g$  must have the same marginal expected utility.

### 3.2 Asymmetric information

Except for the policy design, the previous section has presented a rather standard strategic bequest model. Parents have all the bargaining power and have full information about their children's cost of providing care. We now turn to the more interesting case where parents do not know their children's type. We then have to add two incentive constraints to the parents' problem. The objective function does not change and the participation constraints continue of course to apply. The parents' problem can then be stated as follows

$$\begin{aligned} \max_{\underline{a}, \underline{a}, \overline{\tau}, \underline{\tau}, s} \quad & P^{as} = w\overline{T} - \pi g - s + (1 - \pi)U(s) + \pi[\lambda H(s + \underline{a} - \underline{\tau} + g) \\ & + (1 - \lambda)H(s + \overline{a} - \overline{\tau} + g)] \\ \text{s.t.} \quad & \underline{\tau} - v(\underline{a}, \underline{\beta}) \geq 0, \\ & \overline{\tau} - v(\overline{a}, \overline{\beta}) \geq 0, \\ & \overline{\tau} - v(\overline{a}, \overline{\beta}) \geq \underline{\tau} - v(\underline{a}, \overline{\beta}), \\ & \underline{\tau} - v(\underline{a}, \underline{\beta}) \geq \overline{\tau} - v(\overline{a}, \underline{\beta}). \end{aligned} \quad (8)$$

This is a rather standard mechanism design problem and one can easily show that we obtain the "usual" pattern of binding incentive and participation constraints. To be precise, the participation constraint of high-cost children is binding so that

$$\underline{\tau} = v(\underline{a}, \underline{\beta}). \quad (9)$$

Furthermore, the incentive constraint of  $\bar{\beta}$ , the low-cost type, is binding. Using (9) this condition can be written as

$$\bar{\tau} = v(\bar{a}, \bar{\beta}) + [v(\underline{a}, \underline{\beta}) - v(\underline{a}, \bar{\beta})], \quad (10)$$

where the term in brackets on the RHS represents the rent of type  $\bar{\beta}$ . In words, because low-cost children can mimic high-cost ones and provide  $\underline{a}$  at a lower cost, they have to receive a transfer exceeding the cost of the care they provide. This is particularly interesting from our perspective because it implies that the utility of the caregivers is no longer exogenously given. Some of them now receive a transfers which puts them above their reservation utility level and this transfer may depend on the LTC policy.

Substituting for the transfers from (9) and (10) into the objective function, the parents' problem can then be rewritten as

$$\begin{aligned} \max_{\bar{a}, \underline{a}, s} \quad P^{as} = & w\bar{T} - \pi g - s + (1 - \pi)U(s) + \pi [\lambda H(s + \underline{a} - v(\underline{a}, \underline{\beta}) + g) \\ & + (1 - \lambda)H(s + \bar{a} - v(\bar{a}, \bar{\beta}) - v(\underline{a}, \underline{\beta}) + v(\underline{a}, \bar{\beta}) + g)]. \end{aligned} \quad (11)$$

The first-order conditions are given by

$$\frac{\partial P^{as}}{\partial \bar{a}} = (1 - \lambda)H'(\bar{m})[1 - v_a(\bar{a}, \bar{\beta})] = 0, \quad (12)$$

$$\frac{\partial P^{as}}{\partial \underline{a}} = \pi \{ \lambda H'(\underline{m})[1 - v_a(\underline{a}, \underline{\beta})] - (1 - \lambda)H'(\bar{m})[v_a(\underline{a}, \underline{\beta}) - v_a(\underline{a}, \bar{\beta})] \} = 0, \quad (13)$$

$$\frac{\partial P^{as}}{\partial s} = -1 + (1 - \pi)U'(s) + \pi[\lambda H'(\underline{m}) + (1 - \lambda)H'(\bar{m})] = 0. \quad (14)$$

From equation (12) we obtain  $1 = v_a(\bar{a}, \bar{\beta})$ , which is the full information condition for  $\bar{\beta}$ , the low-cost type; see equation (5). This is the traditional no distortion at the top result that, given the quasi-linearity of the utility function, does not only apply to the *rule*, but also to the actual *level* of care  $\bar{a}$ , which is the same as in the full information solution and continues to be independent of  $g$ .

Turning to  $\underline{a}$ , rearranging (13) yields

$$[1 - v_a(\underline{a}, \underline{\beta})] = \frac{(1 - \lambda)H'(\bar{m})}{\lambda H'(\underline{m})} [v_a(\underline{a}, \underline{\beta}) - v_a(\underline{a}, \bar{\beta})] > 0, \quad (15)$$

where we have used  $v_{a\beta} < 0$ , which implies  $\Delta v_a = v_a(\underline{a}, \underline{\beta}) - v_a(\underline{a}, \bar{\beta}) > 0$ . Consequently we have  $v_a(\underline{a}, \underline{\beta}) < 1$  implying a downward distortion for  $\underline{a}$ . Intuitively,  $\Delta v_a > 0$

accounts for the fact that the rent of the low-cost type increases with  $\underline{a}$ . The downward distortion allows parents to mitigate this rent. Equation (15) also implies that  $\underline{a}$  depends on  $g$ , as well as the bequests left to both types, according to (9) and (10). For  $g = 0$ , the solution to the parents' problem yields the *laissez-faire* allocation.

We now turn to the government's problem. Since the parents no longer have full information, the low-cost children now have a positive utility level. Moreover, their utility is affected by the LTC policy via its impact on the parents' optimization problem. Consequently, the relative weight of children in social welfare is now relevant. When this weight is strictly positive, the LTC policy strikes a balance between providing insurance coverage to parents and the concern for the wellbeing of the caregivers.

The government's problem is given by

$$\begin{aligned} \max_g \quad G^{as} = & \alpha \{ w\bar{T} - \pi g - s + (1 - \pi)U(s) + \pi [\lambda H(s + \underline{a} - v(\underline{a}, \underline{\beta}) + g) \\ & + (1 - \lambda)H(s + \bar{a} - v(\bar{a}, \bar{\beta}) - [v(\underline{a}, \underline{\beta}) - v(\underline{a}, \bar{\beta})] + g)] \} \\ & + (1 - \alpha)\pi(1 - \lambda)[v(\underline{a}, \underline{\beta}) - v(\underline{a}, \bar{\beta})], \end{aligned} \quad (16)$$

where  $\bar{a}$ ,  $\underline{a}$ , and  $s$  are determined by the solution to the parents' problem. Observe that parents' utility is expressed as the solution to the reformulated problem (11) which is an unconstrained optimization where the relevant IC and participation constraints have been substituted into the objective function.

Using the envelope theorem according to which we can neglect the derivatives of parents' utility with respect to  $\underline{a}$ ,  $\bar{a}$ , and  $s$ , the FOC is given by<sup>17</sup>

$$\begin{aligned} \frac{\partial G^{as}}{\partial g} = & \alpha \{ -\pi + \pi[\lambda H'(\underline{m}) + (1 - \lambda)H'(\bar{m})] \} \\ & + \pi(1 - \lambda)(1 - \alpha)[v_a(\underline{a}, \underline{\beta}) - v_a(\underline{a}, \bar{\beta})] \frac{\partial \underline{a}}{\partial g} = 0. \end{aligned} \quad (17)$$

Recall that the term  $\Delta v_a = v_a(\underline{a}, \underline{\beta}) - v_a(\underline{a}, \bar{\beta})$  is positive. Observe that the children's utility does not directly depend on  $s$  and that  $\bar{a}$  does not depend on  $g$ , nor does the high-costs children's utility, which is always equal to zero, the exogenous reservation utility. Consequently, the only behavioral response to the LTC policy that is relevant in

<sup>17</sup>We assume that the second order condition holds.

(17) is  $\partial \underline{a} / \partial g$  which measures how  $g$  affects the level of care provided by the high-cost children. The sign of this expression will in turn determine how the government transfer affects the caregivers' rent.

Consider first the case where the caregivers' utility is not included in social welfare, that is  $\alpha = 1$ . Using the parent's FOC, expression (17) can be written as

$$U'(s) = [\lambda H'(\underline{m}) + (1 - \lambda)H'(\bar{m})] = 1.$$

This is the same *rule* as under full information, as given by (7). In both cases we have  $U'(s) = 1$  so that the *level* of  $s$  is also the same. However, the levels of  $m$  will differ from the full information solution, which in turn implies that the *level* of  $g$  will also in general be different, even though the rule is the same.

We use superscripts  $f$  and  $as$  to refer to the solutions to the full information and asymmetric information problems, specified by (6) and (16) respectively. Suppose that  $g^{as} = g^f$ . Then,  $\lambda H'(\underline{m}^{as}) + (1 - \lambda)H'(\bar{m}^{as}) > 1$ , because  $\underline{m}^{as} < \underline{m}^f$  (as  $\underline{a}$  is distorted downward) and  $\bar{m}^{as} < \bar{m}^f$  (as low-cost children receive a positive rent); consequently we must have  $g^{as} > g^f$ : the optimal level of LTC benefits is larger under asymmetric information than under full information. Intuitively,  $g$  is higher to partially compensate for the downward distortion in  $a$  that parents create to mitigate children's rents.

Let us now turn to the case where  $\alpha < 1$ , which includes the utilitarian case where  $\alpha = 1/2$ . In this case  $g$  is no longer solely determined to provide insurance to parents. The optimal LTC policy also accounts for the impact of  $g$  on informal care and thus on children's utility (rents). Roughly speaking, when  $\partial \underline{a} / \partial g > 0$  one can expect that the effect described for  $\alpha = 1$  is reinforced by the effect of  $g$  on children's rents. Since rents increase in  $\underline{a}$ , increasing  $g$  increases rents. In this case we have  $g^{as} > g^f$ . Conversely, when  $\partial \underline{a} / \partial g < 0$ , the two effects go in opposite directions. Either way this discussion shows that as soon as  $\alpha < 1$  the results will crucially depend on the sign of  $\partial \underline{a} / \partial g$ . The study of this sign requires a closer look at the comparative statics of the parents' problem under asymmetric information. The following lemma is established in Appendix A.1.

**Lemma 1** *When the parents' utility in case of dependence  $H(m)$  exhibits DARA (decreasing absolute risk aversion) we have  $\partial \underline{a} / \partial g < 0$ ; when  $H(m)$  exhibits IARA (in-*

creasing absolute risk aversion) we have  $\partial \underline{a} / \partial g > 0$ .

This lemma implies that the effect of  $g$  (via the parents' problem) on the level of care provided by the high-cost children depends on the parents' attitude towards risk. Intuitively our results can be understood as follows. With DARA, as  $g$  increases, parents become *less* risk averse. Then, reducing  $m$  in the bad state of nature becomes less costly for them, and distorting  $\underline{a}$  downwards becomes more attractive. The case with IARA is exactly symmetrical. Note that empirically DARA appears to receive more support (Friend and Blume, 1975).

Using the FOC of the parents with respect to  $s$ , (14), equation (17) implies that, whenever  $\alpha < 1$

$$U'(s) < 1 < E_{\beta}(H'(m)) \quad \text{if} \quad \partial \underline{a} / \partial g < 0,$$

and

$$U'(s) > 1 > E_{\beta}(H'(m)) \quad \text{if} \quad \partial \underline{a} / \partial g > 0.$$

Intuitively, the utility of dependent parents is distorted down with respect to the full information case under DARA. In this case, providing full insurance against the risk of dependence would push parents to cut the utility of low-cost children (by distorting down  $\underline{a}$ ). From the perspective of social welfare, there is then a tradeoff between insurance and children's utility, leading to less than full insurance. Accordingly, parents have an incentive to save more and this increases their consumption if healthy. Under IARA these effects are reversed.

Using equation (17) and Lemma 1 we can study the effect of  $\alpha$  on  $g^{as}(\alpha)$ . For instance, we can compare the utilitarian level  $g^{as}(1/2)$  with  $g^{as}(1)$ , the level achieved when children are not accounted for in the social welfare function. With DARA we know from Lemma 1 that  $\underline{a}$  decreases as  $g$  increases, which in turn implies that the utility of low-cost children decreases; recall that  $\Delta v_a > 0$ . Consequently equation (17) evaluated at  $g^{as}(1)$  is negative so that  $g^{as}(1/2) < g^{as}(1)$ . Under IARA these effects are reversed and we obtain  $g^{as}(1/2) > g^{as}(1)$ . This result also goes through for intermediate levels



of  $\alpha$ . In Appendix A.2 we show that

$$\frac{\partial g}{\partial \alpha} = \frac{\pi^{\frac{(1-\lambda)}{\alpha}} [v_a(\underline{a}, \underline{\beta}) - v_a(\underline{a}, \bar{\beta})] \frac{\partial \underline{a}}{\partial g}}{SOC_g}, \quad (18)$$

where  $SOC_g$  denotes the second derivative of (16) with respect to  $g$ . Since  $\Delta v_a$  is positive, (18) has the opposite sign of  $\partial \underline{a} / \partial g$ . Accordingly, under DARA  $g$  increases in  $\alpha$ , while it decreases as  $\alpha$  increases under IARA.

To sum up, under DARA  $g$  increases in  $\alpha$ , while  $\underline{a}$  (chosen by parents) is decreasing in  $g$ .<sup>18</sup> Under IARA, on the other hand,  $g$  decreases in  $\alpha$ , while  $\underline{a}$  increases with  $g$ . Consequently, in either case  $\underline{a}$  decreases in  $\alpha$ , and thus increases in  $(1 - \alpha)$ , the children's weight in social welfare. This implies that the larger the weight of children in the social welfare function, the larger the optimal average level of informal care.<sup>19</sup> This apparently counter-intuitive result is due to the exchange motive behind family help. Under this motive, higher reliance on the family for the provision of long-term care implies higher rents for children. The optimal policy implies that the high-cost children will provide more informal care than in the *laissez-faire*, but they will be compensated by higher bequests. This in turn will "spill over" to the low cost children via the incentive constraint and they will be better off than in the *laissez-faire*.

The results obtained in this section are summarized in the following proposition.

**Proposition 1** *Consider the case where the children's cost of providing care is not observable and where public policy is restricted to a uniform LTC benefit  $g$  financed by a lump-sum tax. Informal care is observable only by parents. The optimal LTC policy is such that:*

- (i) *The risk of having children with a high cost of providing care is not fully insured;*
- (ii) *If children's utility has no weight in social welfare, parents are fully insured against dependence. This is achieved through a uniform benefit that is larger than in the full information case.*
- (iii) *If the weight of children in social welfare is strictly positive, and the parents' utility in case of dependence  $H(m)$  exhibits DARA, parents are less than fully insured*

<sup>18</sup> Parents' choices do not directly depend on  $\alpha$ .

<sup>19</sup> Recall that  $\bar{a}$  does not depend on  $\alpha$ , so that average care moves in the same direction as  $\underline{a}$ .

against dependence. The uniform benefit decreases with the weight of children in social welfare.

(iv) If the weight of children in social welfare is strictly positive, and  $H(m)$  exhibits IARA, parents are more than fully insured against dependence. The uniform benefit is higher than under full information and it increases with the weight of children in social welfare.

(v) In either case the average level of care provided by children increases with their weight in social welfare.

## 4 Nonlinear policies

We now consider nonlinear policies under which the level of LTC transfers  $g$  can be conditioned on  $\tau$ , which is publicly observable.<sup>20</sup> We continue to assume that  $a$  is observable only to parents. The underlying problem presents methodological challenges because we have to deal with a “nested” principal-agent problem.<sup>21</sup> The LTC policy can screen for the children’s cost parameter  $\beta$ , but only indirectly via the parents. The latter do not observe their child’s  $\beta$  either but since they observe informal care, they have superior information.

We proceed exactly like in the previous section. We start with the full information solution and then concentrate on the case where neither the parents nor the government can observe the children’s type  $\beta$ .<sup>22</sup> Parents observe  $a$  but the government does not. The policy we study consists of a menu of LTC benefits and bequests pairs. Under asymmetric information parents offer a menu of bequests and informal care pairs, taking

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<sup>20</sup>In practice, *inter vivos* gifts and bequests (estates) are already taxed albeit for different reasons; see Cremer and Pestieau (2006). It is well known that taxes on wealth transfer are subject to evasion and or avoidance. However, this consists mainly in anticipating transfers (there is an exemption for a certain amount of gifts), so much that the estate (or inheritance) tax is often referred to as the “tax on sudden death”. For our purpose the distinction of gifts and transfers is not relevant (only the implementation changes). Consequently the assumption that  $g$  can be conditioned on  $\tau$  does not require more information than policies which are already in place.

<sup>21</sup>The problem considered by Guesnerie and Laffont (1978) has a similar structure. They analyze nonlinear taxation of a monopolist that in turn uses nonlinear pricing.

<sup>22</sup>The “intermediate” case where only parents have full information is also of some interest. However, since the insight it provides is not directly related to our main results, we restrict ourselves to presenting it in an Appendix.

into account the corresponding government transfers. We will also continue to assume that the government cannot make any *direct* transfer to children.

Observe that, while we study a mechanism design problem, the policy can be implemented by a suitably designed mix of LTC benefits and taxes. The mix depends on the precise timing and more specifically on whether  $\tau$  is interpreted as a gift or a bequest. So far we have been agnostic about this because it was of no relevance. When  $\tau$  is a gift and thus precedes the public transfer  $g$ , the solution can be implemented simply by a function  $g(\tau)$  conditioning LTC benefits on  $\tau$ . When  $\tau$  is a bequest, which by definition occurs after  $g$  is consumed, we can condition LTC on a reported (planned)  $\tau$ , but then we have to make sure that parents stick to one of the pairs  $(\underline{\tau}, \underline{g})$  or  $(\bar{\tau}, \bar{g})$ . In particular we have to prevent parents from picking a pair, but then leaving a larger bequest (in order to “buy” more care). This can be done by a nonlinear tax on bequest which is prohibitively large when  $\tau$  deviates from the one associated with the level of public LTC consumed. In practice this means that “excess” public transfers can be recovered from an individual’s bequest.

#### 4.1 Full information solution

In this section, we assume that both parents and the government have full information concerning the children’s types. Transfer  $\tau$  are also publicly observable. However,  $a$  is observed by parents only. The government sets  $\bar{g}, \underline{g}, \bar{\tau}, \underline{\tau}$ , anticipating the choices of the parents. Parents choose  $s$  *ex ante* to maximize their expected utility, and set  $a$  such that  $\tau - v(a, \beta) = 0$ ; we can thus define  $a^f(\tau, \beta)$  as the solution to this equation. We have

$$\begin{aligned}\frac{\partial a^f}{\partial \tau} &= \frac{1}{v_a}, \\ \frac{\partial a^f}{\partial \beta} &= -\frac{v_\beta}{v_a} > 0.\end{aligned}$$

The government now maximizes

$$\begin{aligned} \max_{\bar{g}, \underline{g}, \bar{\tau}, \underline{\tau}} \quad & G^{ff} = w\bar{T} - \pi(\lambda \underline{g} + (1 - \lambda)\bar{g}) - s + (1 - \pi)U(s) \\ & + \pi \left[ \lambda H(s + a^f(\underline{\tau}, \underline{\beta}) - \underline{\tau} + \underline{g}) + (1 - \lambda)H(s + a^f(\bar{\tau}, \bar{\beta}) - \bar{\tau} + \bar{g}) \right]. \end{aligned} \quad (19)$$

The FOCs of the government are

$$\begin{aligned} \frac{\partial G^{ff}}{\partial \bar{g}} &= -\pi(1 - \lambda) + \pi(1 - \lambda)H'(\bar{m}) = 0, \\ \frac{\partial G^{ff}}{\partial \underline{g}} &= -\pi\lambda + \pi\lambda H'(\underline{m}) = 0, \\ \frac{\partial G^{ff}}{\partial \bar{\tau}} &= \pi(1 - \lambda)H'(\bar{m}) \left[ \frac{\partial a^f(\bar{\tau}, \bar{\beta})}{\partial \bar{\tau}} - 1 \right] = 0, \end{aligned} \quad (20)$$

$$\frac{\partial G^{ff}}{\partial \underline{\tau}} = \pi\lambda H'(\underline{m}) \left[ \frac{\partial a^f(\underline{\tau}, \underline{\beta})}{\partial \underline{\tau}} - 1 \right] = 0, \quad (21)$$

and parents choose  $s$  so that

$$\frac{\partial G^{ff}}{\partial s} = -1 + (1 - \pi)U'(s) + \pi[\lambda H'(\underline{m}) + (1 - \lambda)H'(\bar{m})] = 0. \quad (22)$$

Combining these equations yields

$$H'(\bar{m}) = H'(\underline{m}) = U(s) = 1,$$

and

$$v_a(\bar{a}, \bar{\beta}) = v_a(\underline{a}, \underline{\beta}) = 1.$$

These expressions have a simple interpretation. With full information, a nonuniform LTC policy can provide full insurance not only against the risk of dependence, but also against the risk of having high-cost children. Informal care  $a$  is set at its efficient level for each type of children. We now turn to the case where neither parents nor the government observe the children's types. The "intermediate" case where only parents have full information is also of some interest. It is presented in Appendix A.3, which can be skipped without affecting the readability of the following sections.

## 4.2 Asymmetric information

When neither the parents nor the government can observe the children's types, the government proposes a menu  $((\bar{\tau}, \bar{g}), (\underline{\tau}, \underline{g}))$ . Before knowing the type of their children, parents set an incentive compatible contract  $((\bar{\tau}, \bar{a}), (\underline{\tau}, \underline{a}))$ . Since the bequest levels are given, the only choice left to parents is to fix the level of  $a$  associated with each option. As long as  $\bar{\tau} > \underline{\tau}$ , parents set these levels of informal care such that the participation constraint of  $\underline{\beta}$  and the incentive constraint of  $\bar{\beta}$  are satisfied. Formally, the levels of  $a$  are defined by

$$\underline{\tau} = v(\underline{a}^{as}, \underline{\beta}),$$

and

$$\bar{\tau} = v(\bar{a}^{as}, \bar{\beta}) + v(\underline{a}^{as}, \underline{\beta}) - v(\underline{a}^{as}, \bar{\beta}).$$

The optimal (nonuniform) LTC policy is then determined by solving the following problem<sup>23</sup>

$$\begin{aligned} \max_{\bar{g}, \underline{g}, \bar{\tau}, \underline{\tau}} \quad & G^{aa} = \alpha \{ w\bar{T} - \pi(\lambda \underline{g} + (1-\lambda)\bar{g}) - s + (1-\pi)U(s) \\ & + \pi [\lambda H(s + \underline{a}^{as} - \underline{\tau} + \underline{g}) + (1-\lambda)H(s + \bar{a}^{as} - \bar{\tau} + \bar{g})] \} \\ & + (1-\alpha)\pi(1-\lambda)[v(\underline{a}^{as}, \underline{\beta}) - v(\underline{a}^{as}, \bar{\beta})] \\ \text{s.t.} \quad & \underline{\tau} = v(\underline{a}^{as}, \underline{\beta}), \end{aligned} \tag{25}$$

$$\bar{\tau} = v(\bar{a}^{as}, \bar{\beta}) + v(\underline{a}^{as}, \underline{\beta}) - v(\underline{a}^{as}, \bar{\beta}), \tag{26}$$

$$\bar{\tau} \geq \underline{\tau}. \tag{27}$$

To solve this problem, we will first ignore constraint (27). We will then verify *ex post* if the solution to the unconstrained problem fulfils this constraint. If this is the case,

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<sup>23</sup> We ignore for time being the IC constraints of the parents, but we show in footnote 25 that they are satisfied by our solution. These IC constraints are given by

$$\lambda H(s + \underline{a}^{as} - \underline{\tau} + \underline{g}) + (1-\lambda)H(s + \bar{a}^{as} - \bar{\tau} + \bar{g}) \geq H(s + \underline{a}^{as} - \underline{\tau} + \underline{g}) \tag{23}$$

$$\lambda H(s + \underline{a}^{as} - \underline{\tau} + \underline{g}) + (1-\lambda)H(s + \bar{a}^{as} - \bar{\tau} + \bar{g}) \geq H(s + a^{as}(\bar{\tau}, \underline{\beta}) - \bar{\tau} + \bar{g}), \tag{24}$$

where  $a^{as}(\bar{\tau}, \underline{\beta})$  is defined by  $\bar{\tau} = v(a^{as}(\bar{\tau}, \underline{\beta}), \underline{\beta})$ . In words, these constraints can be explained as follows. Since parents have private information on the level of informal care, *ex ante* they could either offer the pooling contract  $(\underline{\tau}, \underline{a}^{as})$  to the children, or alternatively the pooling contract  $(\bar{\tau}, a^{as}(\bar{\tau}, \underline{\beta}))$ . To be incentive compatible for the parents, these must prefer (*ex ante*) the menu  $((\bar{\tau}, \bar{a}), (\underline{\tau}, \underline{a}))$  to any of these pooling contracts.

we can indeed ignore the constraint. If not a pooling equilibrium emerges. Substituting (25) and (26) in the objective function, the problem can be rewritten as

$$\begin{aligned} \max_{\bar{g}, \underline{g}, \bar{a}, \underline{a}} \quad G^{aa} = & \alpha \{ w\bar{T} - \pi(\lambda \underline{g} + (1 - \lambda)\bar{g}) - s + (1 - \pi) U(s) \\ & + \pi \lambda H(s + \underline{a} - v(\underline{a}, \underline{\beta}) + \underline{g}) \\ & + \pi(1 - \lambda) H(s + \bar{a} - v(\bar{a}, \bar{\beta}) - v(\underline{a}, \underline{\beta}) + v(\underline{a}, \bar{\beta}) + \bar{g}) \} \\ & + (1 - \alpha) \pi (1 - \lambda) [v(\underline{a}, \underline{\beta}) - v(\underline{a}, \bar{\beta})]. \end{aligned}$$

The FOCs are given by

$$\frac{\partial G^{aa}}{\partial \bar{g}} = -\pi(1 - \lambda) + \pi(1 - \lambda) H'(\bar{m}) = 0, \quad (28)$$

$$\frac{\partial G^{aa}}{\partial \underline{g}} = -\pi \lambda + \pi \lambda H'(\underline{m}) = 0, \quad (29)$$

$$\frac{\partial G^{aa}}{\partial \bar{a}} = \pi(1 - \lambda) H'(\bar{m}) [1 - v_a(\bar{a}, \bar{\beta})] = 0, \quad (30)$$

$$\frac{\partial G^{aa}}{\partial \underline{a}} = \pi \alpha \lambda H'(\underline{m}) [1 - v_a(\underline{a}, \underline{\beta})] - \pi(1 - \lambda) [\alpha H'(\bar{m}) - (1 - \alpha)] [v_a(\underline{a}, \underline{\beta}) - v_a(\underline{a}, \bar{\beta})] = 0. \quad (31)$$

Conditions (28) and (29), combined with the parents' FOC with respect to savings imply that

$$U'(s) = H'(\underline{m}) = H'(\bar{m}) = 1.$$

Then, we have  $\underline{m} = \bar{m}$  implying that, under asymmetric information, the optimal nonuniform LTC insurance scheme provides full insurance not only against the risk of dependence, but also against the uncertainty associated with informal care. This is in stark contrast with the results obtained with a uniform policy where full insurance could not be achieved.

Informal care is set at its full information level for low-cost children, as it is shown in (30), which implies  $v_a(\bar{a}^*, \bar{\beta}) = 1$ . Conversely, the optimal level of informal care provided by high-cost children is distorted. Combining (30) with (29) and (31) shows that an interior solution for  $\underline{a}$  is determined by

$$\lambda \alpha [1 - v_a(\underline{a}, \underline{\beta})] - (1 - \lambda) (2\alpha - 1) [v_a(\underline{a}, \underline{\beta}) - v_a(\underline{a}, \bar{\beta})] = 0. \quad (32)$$

Since  $v_a$  decreases in  $\beta$ , the direction of the distortion depends on the sign of  $(2\alpha - 1)$ . If children have a lower weight than parents in the social welfare function ( $\alpha > 1/2$ ), then  $\underline{a}$  is distorted downward. If parents and children have the same weight, ( $\alpha = 1/2$ ), there will be no distortion of informal care. Intuitively, in this case the rent is purely a transfer, since the social marginal utility of consumption is the same for children and parents. Finally, if children have a higher weight than parents, there will be an upward distortion of informal care.<sup>24</sup>

The optimal LTC transfer is higher the lower the level of bequests (and the lower the level of informal care). To see this, consider the allocation characterized by (28)–(31). Since individuals are fully insured,  $\underline{m} = \overline{m}$ , which implies

$$g(\overline{\tau}) \leq g(\underline{\tau}) \iff \overline{a} - \overline{\tau} \geq \underline{a} - \underline{\tau}.$$

Using the IC and PC of the children, this can be rewritten as

$$g(\overline{\tau}) \leq g(\underline{\tau}) \iff \overline{a} - v(\overline{a}, \overline{\beta}) \geq \underline{a} - v(\underline{a}, \overline{\beta}),$$

which is always true since  $\overline{a} = \arg \max_x x - v(x, \overline{\beta})$ . Then, the optimal separating policy  $g(\tau)$  implies higher transfers to the parents of high-cost children, in order to compensate them for the lower level of informal care they receive.<sup>25</sup>

The allocation characterized above is a solution to the government's problem only if it satisfies (27). Denote  $\underline{a}^*$  and  $\overline{a}^*$  the solutions to (32) and (30). Differentiating (32) shows that  $\underline{a}^*$  always decreases in  $\alpha$  (and thus increases in children's weight  $1 - \alpha$ ),

<sup>24</sup>These results do depend on the assumption that lump-sum transfers between generations are ruled out. If such transfers were available, the optimal policy would be simply to maximize total surplus (the "size of the cake") and then select one of the corners depending on the weights. It is the absence of lump-sum transfers which makes rents costly, a property which is a cornerstone of contract theory.

<sup>25</sup>Because the solution implies  $\underline{m} = \overline{m}$ , it immediately follows that the parents' IC constraint (23) stated in Footnote 23 is satisfied (as equality). Furthermore, by definition we have  $\overline{\tau} = v(a^{as}(\overline{\tau}, \underline{\beta}), \underline{\beta})$ . Substituting into (26) yields

$$v(\overline{a}^{as}, \overline{\beta}) - v(a^{as}(\overline{\tau}, \underline{\beta}), \underline{\beta}) = v(\underline{a}^{as}, \overline{\beta}) - v(\underline{a}^{as}, \underline{\beta}). \quad (33)$$

Since  $v_{a\beta} < 0$  we have  $v(\overline{a}^{as}, \overline{\beta}) - v(\overline{a}^{as}, \underline{\beta}) < v(\underline{a}^{as}, \overline{\beta}) - v(\underline{a}^{as}, \underline{\beta})$ , which together with (33) implies  $v(\overline{a}^{as}, \overline{\beta}) - v(\overline{a}^{as}, \underline{\beta}) < v(\underline{a}^{as}, \overline{\beta}) - v(a^{as}(\overline{\tau}, \underline{\beta}), \underline{\beta})$  so that  $a^{as}(\overline{\tau}, \underline{\beta}) < \overline{a}^{as}$ . Consequently,  $s + a^{as}(\overline{\tau}, \underline{\beta}) - \overline{\tau} + \overline{g} < \overline{m} = s + \overline{a}^{as} - \overline{\tau} + \overline{g}$  which shows that the second of the parents' IC constraint (24) is also satisfied.

irrespective of the degree of risk aversion of the parents.<sup>26</sup> Since  $\bar{a}^*$  does not depend on  $\alpha$ , this implies that like under a uniform policy the average level of care increases with the children's weight in social welfare. The intuition underlying this result is also the same as in the uniform case. Children are paid for the extra care they provide through higher transfers. Under the optimal policy high-cost children provide more care than in the *laissez-faire*, but they will be compensated by higher bequests or gifts. This in turn increases the informational rents of the low-cost children.

If children have no weight in the social welfare function ( $\alpha = 1$ ), then  $\underline{a}$  is distorted downwards, so that  $\underline{a}^* < \bar{a}^*$ . If  $\alpha = 0$ , then the LHS of (32) is always positive. In this case there is a corner solution with  $\underline{a}^* = a^{max} > \bar{a}^*$ , where  $a^{max}$  is the maximum level of informal care that can be provided by children. Then, there exist a threshold  $\hat{\alpha} = (1 - \lambda)/(2 - \lambda) < 1/2$  such that the optimal policy implies  $\underline{a} = \underline{a}^* \leq \bar{a}^*$  if and only if  $\alpha \geq \hat{\alpha}$ .<sup>27</sup> Using (25) and (26) it follows that this solution satisfies constraint (27).

When  $\alpha < \hat{\alpha}$  we have  $\underline{a}^* > \bar{a}^*$ , which violates constraint (27), since

$$\tau = v(\underline{a}^*, \underline{\beta}) > v(\bar{a}^*, \bar{\beta}) + v(\underline{a}^*, \underline{\beta}) - v(\underline{a}^*, \bar{\beta}) = \bar{\tau},$$

whenever  $\underline{a}^* > \bar{a}^*$ . In this case, the optimal policy consists in a pooling contract  $\{\tau^p, g^p\}$ . Observe that, since  $\hat{\alpha} < 1/2$ , pooling may occur only when children receive a sufficiently larger weight than parents in social welfare.

To complete the analysis let us now determine this pooling equilibrium. Under this contract parents set  $a^p$  such that  $\tau^p = v(a^p, \underline{\beta})$  and the government's problem can be written as

$$\begin{aligned} \max_{g, a} \quad G^{aap} = & \alpha \{ w\bar{T} - \pi g - s + (1 - \pi) U(s) + \pi H(s + a - v(a, \underline{\beta}) + g) \} \\ & + (1 - \alpha) \pi (1 - \lambda) [v(a, \underline{\beta}) - v(a, \bar{\beta})]. \end{aligned}$$

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<sup>26</sup>Differentiating (32), one obtains that the sign of  $\partial \underline{a}^* / \partial \alpha$  is equal to the sign of

$$\lambda [1 - v_a(\underline{a}, \underline{\beta})] - 2(1 - \lambda) [v_a(\underline{a}, \underline{\beta}) - v_a(\underline{a}, \bar{\beta})],$$

which is always negative under (32) since  $(2\alpha - 1)/\alpha \leq 1$  for all  $\alpha \leq 1$ .

<sup>27</sup>The threshold  $\hat{\alpha}$  is the value of  $\alpha$  such that (32) is satisfied for  $\underline{a} = \bar{a}^*$ .



The FOCs are given by

$$\frac{\partial G^{aap}}{\partial g} = -\pi + \pi H' = 0, \quad (34)$$

$$\frac{\partial G^{aap}}{\partial a} = \pi \alpha H' [1 - v_a(a, \underline{\beta})] + \pi (1 - \lambda) (1 - \alpha) [v_a(a, \underline{\beta}) - v_a(a, \bar{\beta})] = 0. \quad (35)$$

Condition (34) yields  $H' = 1$ . This, combined with the FOC of the parents, implies  $1 = U'(s) = H'(m)$  so that we continue to have full insurance against both the risk of dependence and the risk of having a high-cost child. Furthermore, (35) can be rewritten as

$$[1 - v_a(a^p, \underline{\beta})] + (1 - \lambda) \frac{(1 - \alpha)}{\alpha} [v_a(a^p, \underline{\beta}) - v_a(a^p, \bar{\beta})] = 0.$$

Differentiating this expression and making use of the second order condition show that  $\partial a^p / \partial \alpha < 0$  so that under the pooling contract informal care continues to increase with the weight of children in the social welfare function.

It may at first be surprising that with nonlinear instruments even the pooling contract performs better than the uniform contract considered in Section 3; in particular, it provides full insurance which the uniform contract does not. In other words, the pooling contract, though effectively uniform is *not* equivalent to the uniform policy characterized in the previous section. This is because with nonlinear instruments the government controls the level of bequest  $\tau$ . This provides an extra instrument through which the government can indirectly control the level of informal care.

The main results of this section are summarized in the following proposition.

**Proposition 2** *Consider the case where the children's cost of providing care is not observable and where LTC benefits  $g$  can be conditioned on the transfer  $\tau$  paid by parents to children in exchange for informal care. Informal care is observable only to parents. The optimal LTC policy may involve a separating or a pooling contract. This policy is such that:*

- (i) *The risk of having high-cost children is fully insured.*
- (ii) *The average level of informal care always increases in the weight of children in social welfare, irrespective of the parents' degree of risk aversion.*

(iii) *A separating contract is optimal if and only if  $\alpha \geq (1 - \lambda)/(2 - \lambda) < 1/2$ . It implies that:*

(a) *Informal care is set at its first best level for the low-cost children.*

(b) *The level of informal care provided by high-cost children is distorted and the direction of the distortion depends on children's weight in the welfare function. It has the same sign as  $(1 - 2\alpha)$  so that a downward (upward) distortion occurs when the weight of the children is lower (higher) than the weight of parents.*

Note that, while with a uniform policy the results crucially depend on parents' risk aversion, this is no longer true when the policy is restricted by informational considerations only, and can be nonuniform. In this case the available policy instruments are sufficiently powerful to ensure that parents are always fully insured, even against the risk of having high-cost children. And since they are fully insured, risk aversion no longer matters. Even more strikingly, the tradeoff between the provision of insurance to parents and the concern for the welfare of the caregivers which drives the results for a uniform policy is no longer relevant under nonuniform policies.

## 5 Conclusion

We study the design of LTC policy when informal care from children to dependent parents is motivated, at least in part, by the prospects of a gift or bequest. Parents do not observe their children's cost of providing care (determined by various factors, including their degree of altruism), but they can commit to a bequests rule specifying a transfer conditional on the level of informal care. The social welfare function is a weighted sum of parents' and children's utility. We show that social LTC insurance affects the exchanges between parents and children and in particular the level of informal care and the distribution of rents.

The optimal *uniform* public LTC insurance depends on the attitude towards risk of parents. Under DARA (decreasing absolute risk aversion) preferences, public LTC insurance exacerbates the distortion of informal care. Consequently, the optimal public

LTC coverage provides less than full insurance. The opposite is true under IARA (increasing absolute risk aversion) preferences. A uniform policy can never insure the risk of having a high-cost child.

A *nonuniform* policy that conditions LTC benefits on bequests provides full insurance even for the risk of having high-cost children. The level of informal care provided by high-cost children is distorted and the direction of the distortion depends on children's weight in the social welfare function.

Interestingly, in the uniform as well as in the nonuniform case, the higher the weight of children in the social welfare function, the higher the optimal average level of informal care. This apparently counter-intuitive result is due to the exchange motive behind family help. Under this motive, higher reliance on the family for the provision of long-term care implies higher rents for children. In our model, crowding out of family help by public care only affects social welfare through its (negative) effect on children's utility. A main lesson that emerges from our analysis is that in an exchange-based setting, social insurance should be designed in order to ensure that dependent elderly have to rely even on high-cost children. This ensures that low-cost children get rewarded for the informal care they provide.

Another major lesson is that, even with *ex ante* identical individuals, the nonuniform policy performs better and is able to provide full insurance against both underlying risks. In other words, even with identical individuals, social LTC should involve some measure of means testing and/or recover part of the benefits received by the elderly from their estate. This result is interesting because means testing is usually justified by redistribution. We have not considered redistributive motive, but they could only be expected to reinforce this result.

Throughout the paper we have remained agnostic about the exact nature of the transfer, gift or bequest, that "pays" for the care, except that we have pointed out that it affects the timing of the underlying game. From an empirical perspective, however, the gift interpretation appears to be more compelling. The literature has found some evidence that *inter vivos* transfers are larger for those children who provide informal care (Norton and Van Houtven, 2006). However, these estimates are often

problematic because of endogeneity problems and because measurement of both informal care and financial transfers within the family is difficult. More reliable estimates require longitudinal data and a step in that direction is taken by Norton *et al.* (2014) who use the 1999 and 2003 waves of National Longitudinal Survey of Mature Women and show that children providing informal care are indeed more likely to receive financial transfers from their parents. More recently, Ciani and Deiana (2017), use data from three waves of the Italian Multipurpose Survey on Family and Childhood Conditions, and show that “parents who helped their adult children in the past are rewarded by a greater likelihood of receiving informal care later in life”.

Considering bequests as payment for care is more problematic because research has shown that bequests are typically divided equally among children (Menchik, 1980; Tomes, 1981). Even when children provide unequal amounts of informal care, bequests tend to be divided equally (Norton and Taylor, 2005). This is true for the US but the argument is even more compelling for most European countries, where equal sharing rules are imposed by law. In any event there are also theoretical arguments that favor *inter vivos* transfers over bequests as payment for informal care. Gifts are more flexible and, as argued by Norton and Van Houtven (2006), “can be adjusted quickly to the amount of care, are less costly than writing a will, and can be kept secret from other family members and the public”.

Our policy recommendations are made under the assumption that the provision of informal care is exchange-based. This hypothesis has received empirical support and appears to apply to certain families. Roughly speaking, “. . . the idea of exchange makes sense for those extended families where an older person has money and needs help, and a younger person has time and needs money”, (Norton *et al.* 2014). But it is clearly not the only behavioral pattern that is relevant. In reality the different types of intra-family relations, based on altruism, norms or selfish exchanges, with and without commitment are likely to coexist.<sup>28</sup> The different studies provide partial and intermediate answers which can provide valuable guidance for the design of social LTC policy, as long as

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<sup>28</sup> Alternative approaches are explored for instance by Cremer and Roeder (2017) Barrigozzi *et al.* (2017), Canta and Pestieau (2014), Ponthière (2014).

interpreted with suitable care and keeping in mind the underlying assumptions.

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## Appendix

### A.1 Proof of Lemma 1

Since  $\bar{a}$  is independent off  $g$ , we can focus on the FOCs (13) and (14) to study the comparative statics with respect to  $g$ . Using subscripts to denote partial derivative, define

$$H = \begin{bmatrix} P_{\underline{a}\underline{a}}^{as} & P_{\underline{s}\underline{a}}^{as} \\ P_{\underline{a}s}^{as} & P_{\underline{s}s}^{as} \end{bmatrix},$$

and

$$D = \begin{bmatrix} -P_{\underline{a}g}^{as} \\ -P_{\underline{s}g}^{as} \end{bmatrix},$$

where

$$P_{\underline{s}\underline{a}}^{as} = P_{\underline{a}s}^{as} = \pi[\lambda H''(\underline{m})(1 - v(\underline{a}, \underline{\beta}) - (1 - \lambda)H''(\bar{m})(v_a(\underline{a}, \underline{\beta}) - v_a(\underline{a}, \bar{\beta}))) = \pi A, \quad (\text{A.1})$$

$$P_{\underline{s}s}^{as} = (1 - \pi)U''(s) + \pi[\lambda H''(\underline{m}) + (1 - \lambda)H''(\bar{m})], \quad (\text{A.2})$$

$$P_{\underline{a}g}^{as} = \pi \left\{ \lambda H''(\underline{m})[1 - v_a(\underline{a}, \underline{\beta})] - (1 - \lambda)H''(\bar{m})[v_a(\underline{a}, \underline{\beta}) - v_a(\underline{a}, \bar{\beta})] \right\} = \pi A, \quad (\text{A.3})$$

$$P_{\underline{s}g}^{as} = \pi[\lambda H''(\underline{m}) + (1 - \lambda)H''(\bar{m})], \quad (\text{A.4})$$

and where

$$A = \lambda H''(\underline{m})[1 - v_a(\underline{a}, \underline{\beta})] - (1 - \lambda)H''(\bar{m})[v_a(\underline{a}, \underline{\beta}) - v_a(\underline{a}, \bar{\beta})]. \quad (\text{A.5})$$

Using Cramer's rule we obtain

$$\frac{\partial \underline{a}}{\partial g} = \frac{\begin{vmatrix} -P_{\underline{a}g}^{as} & P_{\underline{s}\underline{a}}^{as} \\ -P_{\underline{s}g}^{as} & P_{\underline{s}s}^{as} \end{vmatrix}}{|H|},$$

where  $|H| > 0$  from the second order condition of the parents' problem.

Substituting from (A.1)–(A.4), evaluating the determinant and simplifying successively yields

$$\begin{aligned} \text{sgn} \left( \frac{\partial \underline{a}}{\partial g} \right) &= \text{sgn} \left( \begin{vmatrix} -A\pi & A\pi \\ \pi[\lambda H''(\underline{m}) + (1 - \lambda)H''(\bar{m})] & (1 - \pi)U''(s) + \pi[\lambda H''(\underline{m}) + (1 - \lambda)H''(\bar{m})] \end{vmatrix} \right) \\ &= \text{sgn}(-\pi A[(1 - \pi)u''(s)]) = \text{sgn}(A). \end{aligned}$$

To sum up we have to study the sign of  $A$  defined by (A.5). Substituting from (15) and rearranging yields

$$A = (1 - \lambda)\Delta v_a \left[ H''(\underline{m}) \frac{H'(\bar{m})}{H'(\underline{m})} - H''(\bar{m}) \right].$$

Because  $\Delta v_a > 0$ , this expression has the same sign as the term in brackets on the RHS.

Consequently we have

$$\begin{aligned} A > 0 & \iff \frac{H''(\underline{m})}{H'(\underline{m})} > \frac{H''(\bar{m})}{H'(\bar{m})} \\ & \iff -\frac{H''(\underline{m})}{H'(\underline{m})} < -\frac{H''(\bar{m})}{H'(\bar{m})}. \end{aligned}$$

Since  $\underline{m} < \bar{m}$  this is true under IARA (Increasing Absolute Risk Aversion), while DARA (Decreasing Absolute Risk Aversion) yields  $A < 0$ .

## A.2 Variation of the uniform policy $g$ with respect to $\alpha$

Recall that  $\underline{a}$  and  $s$  are determined by the parents. Consequently they are not *directly* affected by  $\alpha$ , but depend indirectly on the weight via its impact on  $g$ . With this in mind, totally differentiating (17) yields

$$\frac{\partial g}{\partial \alpha} = \frac{\pi[1 - \lambda H'(\underline{m}) + (1 - \lambda)H'(\bar{m})] + \pi(1 - \lambda)[v_a(\underline{a}, \underline{\beta}) - v_a(\underline{a}, \bar{\beta})] \frac{\partial \underline{a}}{\partial g}}{SOC_g},$$

where  $SOC_g$  denotes the second derivative of (16) with respect to  $g$ . Using (17), we get

$$\frac{\partial g}{\partial \alpha} = \frac{\pi \frac{1-\alpha}{\alpha} [v_a(\underline{a}, \underline{\beta}) - v_a(\underline{a}, \bar{\beta})] \frac{\partial \underline{a}}{\partial g} + \pi(1 - \lambda)[v_a(\underline{a}, \underline{\beta}) - v_a(\underline{a}, \bar{\beta})] \frac{\partial \underline{a}}{\partial g}}{SOC_g},$$

which is equivalent to (18).

## A.3 Only parents have full information

If the government does not observe  $\beta$ , but parents do, the problem of the government is given by

$$\begin{aligned} \max_{\underline{g}, \underline{g}, \bar{\tau}, \underline{\tau}} \quad & G^{af} = w\bar{T} - \pi(\lambda \underline{g} + (1 - \lambda)\bar{g}) - s + (1 - \pi)U(s) \\ & + \pi \left[ \lambda H(s + a^f(\underline{\tau}, \underline{\beta}) - \underline{\tau} + \underline{g}) + (1 - \lambda)H(s + a^f(\bar{\tau}, \bar{\beta}) - \bar{\tau} + \bar{g}) \right] \\ \text{s.t.} \quad & a^f(\underline{\tau}, \underline{\beta}) - \underline{\tau} + \underline{g} \geq a^f(\bar{\tau}, \underline{\beta}) - \bar{\tau} + \bar{g}, \\ & a^f(\bar{\tau}, \bar{\beta}) - \bar{\tau} + \bar{g} \geq a^f(\underline{\tau}, \bar{\beta}) - \underline{\tau} + \underline{g}, \end{aligned} \tag{A.6}$$

where we added the relevant *IC* constraints to (19). We shall assume that (A.6), the constraint from the low-cost type to the high-cost type is binding. This constraint is effectively violated at the full information solution characterized in Section 4.1. To see this recall that this solution implies

$$a^f(\bar{\tau}, \bar{\beta}) - \bar{\tau} + \bar{g} = a^f(\underline{\tau}, \underline{\beta}) - \underline{\tau} + \underline{g} < a^f(\underline{\tau}, \bar{\beta}) - \underline{\tau} + \underline{g},$$

so that

$$\bar{g} < \underline{g} + \bar{\tau} - \underline{\tau} + a^f(\underline{\tau}, \bar{\beta}) - a^f(\bar{\tau}, \bar{\beta}),$$

which violates condition (A.6). Substituting the incentive constraint into  $G^{af}$  the government's problem can then be rewritten as

$$\begin{aligned} \max_{\underline{g}, \bar{\tau}, \underline{\tau}} \quad G^{af} = & w\bar{T} - \pi(\underline{g} + (1 - \lambda) (\bar{\tau} - \underline{\tau} + a^f(\underline{\tau}, \bar{\beta}) - a^f(\bar{\tau}, \bar{\beta}))) - s + (1 - \pi)U(s) \\ & + \pi\lambda H(s + a^f(\underline{\tau}, \underline{\beta}) - \underline{\tau} + \underline{g}) + \pi(1 - \lambda)H(s + a^f(\underline{\tau}, \bar{\beta}) - \underline{\tau} + \underline{g}). \end{aligned}$$

The FOCs are given by

$$\frac{\partial G^{af}}{\partial \underline{g}} = -\pi + \pi [\lambda H'(\underline{m}) + (1 - \lambda)H'(\bar{m})] = 0, \quad (\text{A.7})$$

$$\frac{\partial G^{af}}{\partial \bar{\tau}} = \pi(1 - \lambda) \left[ \frac{\partial a^f(\bar{\tau}, \bar{\beta})}{\partial \bar{\tau}} - 1 \right] = 0, \quad (\text{A.8})$$

$$\frac{\partial G^{af}}{\partial \underline{\tau}} = \pi\lambda H'(\underline{m}) \left[ \frac{\partial a^f(\underline{\tau}, \underline{\beta})}{\partial \underline{\tau}} - 1 \right] + \pi(1 - \lambda) (H'(\bar{m}) - 1) \left[ \frac{\partial a^f(\underline{\tau}, \bar{\beta})}{\partial \underline{\tau}} - 1 \right] = 0. \quad (\text{A.9})$$

Recall that  $a^f(\underline{\tau}, \bar{\beta}) > a^f(\underline{\tau}, \underline{\beta})$ , which implies  $\bar{m} = s + a^f(\underline{\tau}, \bar{\beta}) - \underline{\tau} + \underline{g} > \underline{m} = s + a^f(\underline{\tau}, \underline{\beta}) - \underline{\tau} + \underline{g}$ . Hence, condition (A.7) implies that  $H'(\bar{m}) < 1$ . Furthermore, using our assumption on  $a^f$  along with (21), which defines  $\underline{\tau}^{ff}$ , we have

$$\frac{\partial a^f(\underline{\tau}^{ff}, \bar{\beta})}{\partial \underline{\tau}} > \frac{\partial a^f(\underline{\tau}^{ff}, \underline{\beta})}{\partial \underline{\tau}} = 1,$$

so that at  $\underline{\tau}^{ff}$  the first term on the RHS of (A.9) is zero while the second term is negative, which in turn implies  $\underline{\tau}^{af} < \underline{\tau}^{ff}$  (from the concavity of the government's problem). This is not surprising. In order to relax the IC constraint, the optimal policy distorts  $\underline{\tau}$  downwards, which leads to a downward distortion on  $\underline{a}$ . Conversely,  $\bar{\tau}$  and  $\bar{a}$  are not distorted; condition (A.8) is identical to its full information counterpart (20).

Combining (A.7) with (22), the first-order condition for parents' saving , yields

$$U'(s) = [\lambda H'(\underline{m}) + (1 - \lambda)H'(\bar{m})] = 1$$

As in the case with uniform transfers, the optimal LTC policy implies full insurance against dependence but, under asymmetric information, it is not possible to provide insurance against the risk of having high-cost children.